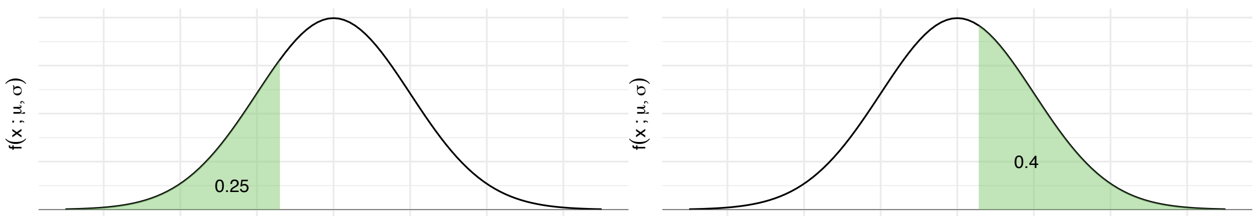
Percentiles

```r
qnorm(0.25, mean = mu, sd = sigma)
```

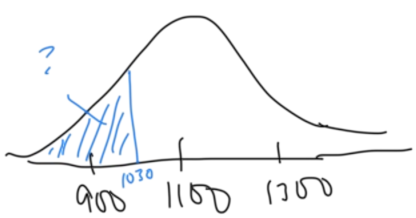


Percentile problems

Suppose SAT scores are Normally distributed with mean 1100 and standard deviation 200.

1. Edward earned a 1030 on his SAT. We will find his percentile using code in two ways.

   a. Draw a picture representing what we want to find. Then write code to find the percentile.

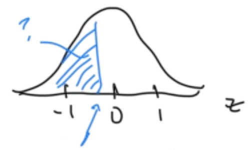


$P(X \le 1030) =$ pnorm(1030, 1100, 200)

$= 0.363.$

   b. Find Edward's z-score. Write another line of code to find the percentile.

$$z = \frac{x-\mu}{\sigma} = \frac{1030-1100}{200} = -0.35.$$



Because $X \sim N(1100, 200)$, $Z \sim N(0,1)$!

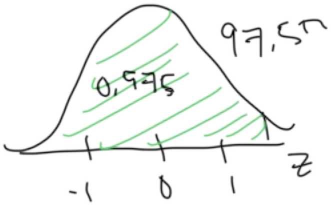
So $P(X \le 1030) \equiv P(Z \le -0.35) =$ pnorm(-0.35, 0, 1) $= 0.363$

2. What is the 97.5th percentile for SAT scores?

   a. Write code to answer this question.

qnorm(0.975, 1100, 200) $\approx$ 1492

b. Write a different line of code to answer this question that involves a z-score.



97.5th percentile of standard normal: qnorm(0.975,0,1) = 1.96

$$1.96 = \frac{x - 1100}{200} \Rightarrow x - 1100 = 200(1.96)$$

$$\Rightarrow x = 1492$$

3. Unrelated to SAT scores: consider the standard normal $N(0, 1)$ distribution. The 25th percentile of this distribution is -0.67. Without doing any work beyond drawing a picture, what is the 75th percentile of the distribution?

? is 75th percentile.

By symmetry, 75th percentile is $x = 0.67$

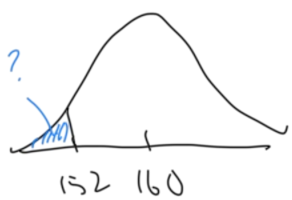
Some practice problems

σ^2, not σ

1. In a law school class, the entering students averaged 160 on the LSAT. The *variance* was 64. The histogram of LSAT scores followed the normal curve reasonable well.

   lets me assume $X \sim N(160, 8)$

   a. About what percentage of the class scored below 152?



Using 68-95, 99.7 rule,

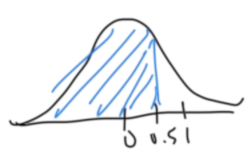
$P(X \leq 152) \approx 1 - (0.5 + 0.34) = 0.16$

   b. One student was 0.5 standard deviations above average on the LSAT. About what percentage of the students had lower scores than he did?

z-score!

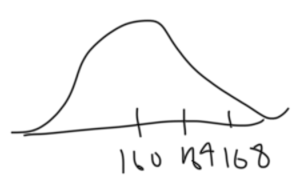
Because we have normality, z-scores $\sim N(0,1)$.

Opt. 1 : pnorm(0.5, 0, 1) = 0.69



Opt. 2 : convert to original units

$$0.5 = \frac{x - 160}{8} \Rightarrow x = 164$$

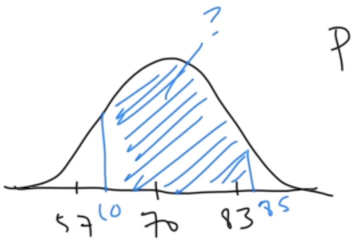


pnorm(164, 160, 8) = 0.69

$X \sim N(70, 13)$

2. Weights of 10-year-old girls are known to be Normally distributed with mean of 70 pounds and standard deviation of 13 pounds. Find the probability that a 10-year-old girl weighs between 60 and 85 pounds in the following two ways. *Optional, but helpful: draw a sketch of the curve and shade in the region of interest.*

   a. Write the probability of interest in $P()$ form. Then write the `R` code necessary to find this probability, and actually execute the code to obtain the probability.



$P(60 \leq X \leq 85)$ = pnorm(85, 70, 13) - pnorm(60, 70, 13)

= 0.655

   b. Confirm your solution in (b) by transforming to z-scores first, then using code again to obtain the probability.

$z_{60} = \frac{60-70}{13} = -0.769$

$z_{85} = \frac{85-70}{13} = 1.154$

since X is Normal, $Z \sim N(0,1)$

$P(-0.769 \leq Z \leq 1.154)$

= pnorm(1.154, 0, 1) - pnorm(-0.769, 0, 1)

= 0.655

3. Consider the same scenario as in 2. Without using any code than what is provided below, find the 60th percentile for the weight of 10-year-old girls.

`qnorm(0.6, mean = 0, sd = 1)` = 0.2533471

$\underbrace{\texttt{mean = 0, sd = 1}}_{N(0,1)}$

60th percentile of N(0,1) is 0.253.

So 60th percentile of N(70,13) is

$0.253 = \frac{x-70}{13} \Rightarrow x = 13(0.253) + 70$

$= 73.289$ pounds