Hypothesis Testing via Randomization

2025-10-20

Where we’re going today

- We will see another kinds of hypotheses for different types of research questions
- Hypothesis testing framework is the same, but will change how we obtain null distribution
- Try to see the big picture

Test of independence

Running example: sex discrimination study

- Note: this study considered sex as binary “male” or “female”, and did not take into consideration gender identities
- Participants in the study were 48 bank supervisors who identified as male and were attending a management institute at UNC in 1972
  - Each supervisor was asked to assume the role of personnel director of a bank
  - Each given a file to judge whether the person in the file should be promoted
  - The files were identical, except half of them indicated that the candidate was male, and the other half were indicated as female
  - Files were randomly assigned to bank managers
  - Experiment or observational study?
- Research question: Are individuals who identify their sex as female discriminated against in promotion decisions made by their managers who identify as male?

Defining hypotheses

Research question: Are individuals who identify their sex as female discriminated against in promotion decisions made by their managers who identify as male?

- What is/are the variables(s) here? What types of variables are they?
- We need to construct hypotheses where H_0 is "status quo" and H_A is the claim researchers have
- H_0: the variables `sex` and `decision` are independent.
  - i.e. any observed difference in promotion rates is due to variability
- H_A: the variables `sex` and `decision` are *not* independent; equally-qualified female personnel are less likely to be promoted than male personnel

Data

For each of the 48 supervisors, the following were recorded:

- The sex of the candidate in the file (male/female)
- The decision (promote/not promote)

sex	not promote	promote	total
female	10	14	24
male	3	21	24
total	13	35	48

- What evidence do we have? What summary statistic(s) would be useful for answering the research question?

Data (cont.)

Conditional probability of getting promoted by sex:

```r
# look at data
discrimination |>
  slice(1:4)
```

```
     sex    decision
1   male     promote
2 female not promote
3   male     promote
4 female     promote
```

```r
discrimination |>
  count(sex, decision) |>
  group_by(sex) |>
  mutate(cond_prob = n/sum(n)) |>
  filter(decision == "promote") |>
  select(-n)
```

sex	decision	cond_prob
female	promote	0.583
male	promote	0.875

- Is the observed difference $\hat{p_{f,obs}} - \hat{p_{m,obs}}$ = -0.2916667 **convincing evidence**? We need to examine variability in the data, assuming H_0 true.
- Let’s set $\alpha = 0.05$

Simulate under null

- Simulating under H_0 means operating in a hypothetical word where `sex` and `decision` are independent.
  - This means that knowing the `sex` of the candidate should have no bearing on the `decision` to promote or not
- We will perform a simulation called a **randomization test**:
  - **Randomly** pair up `decision` and `sex` outcome pairs
  - Randomly assigning a decision to each person would be equivalent to a world in which the bankers' `decision` had been independent of candidate's `sex` (i.e. if H_0 true)

Randomization test

sex	not promote	promote	total
female	10	14	24
male	3	21	24
total	13	35	48

- Write down “promote” on 35 cards and “not promote” on 13 cards. Repeat the following:
  - Thoroughly shuffle these 48 cards.
  - Deal out a stack of 24 cards to represent males, and the remaining 24 cards to represent females
    - This is how we simulate under H_0
  - Calculate the proportion of “promote” cards in each stack, $\hat{p}_{f,sim}$ and $\hat{p}_{m,sim}$
  - Calculate and record the difference $\hat{p}_{f,sim} - \hat{p}_{m,sim}$ (order of difference doesn’t matter so long as you are consistent)

Randomization test (activity)

Try it!

Randomization test (code)

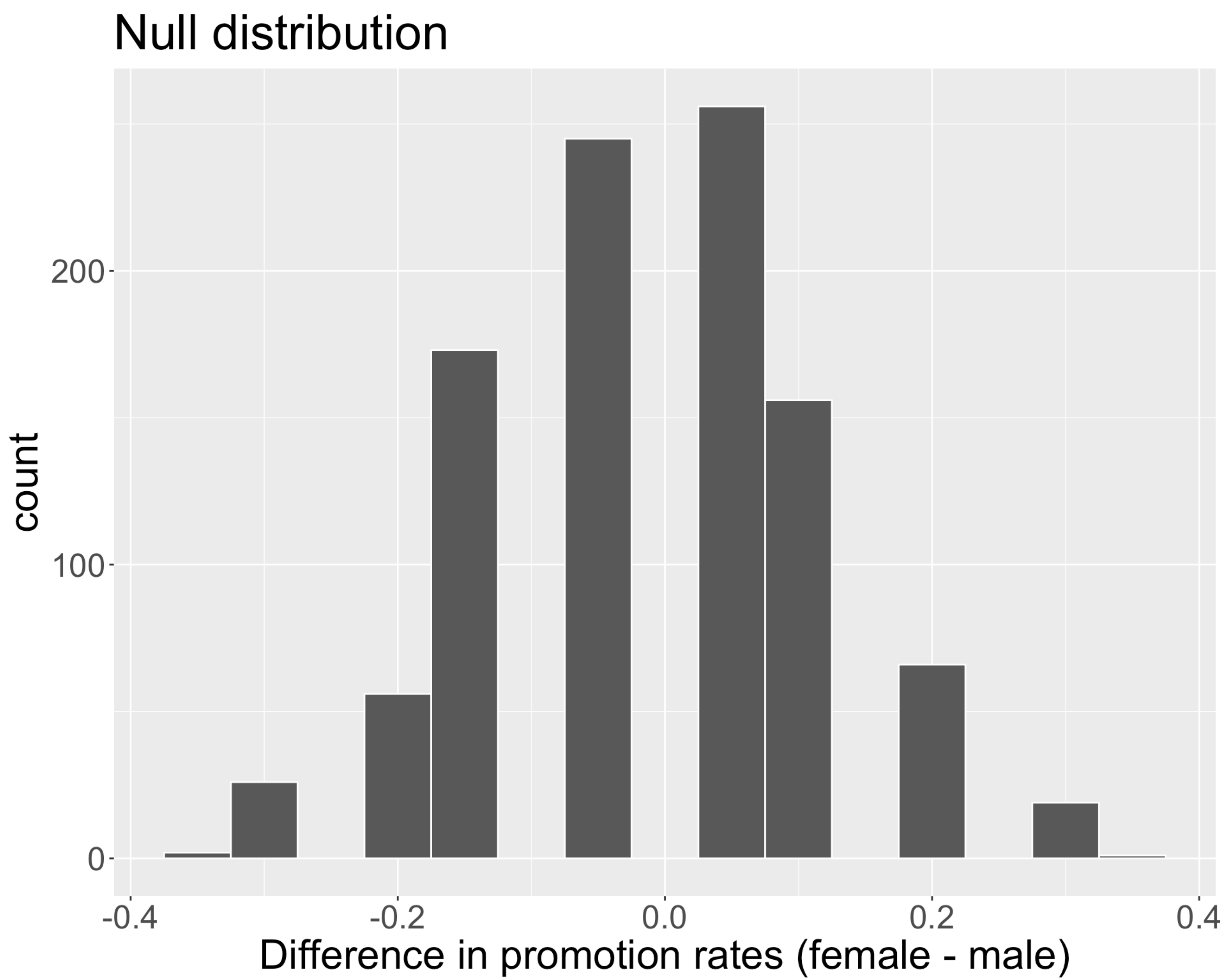
```r
set.seed(100)
n <- nrow(discrimination)
n_f <- sum(discrimination$sex == "female")
n_m <- sum(discrimination$sex == "male")
decisions <- discrimination$decision
B <- 1000
diff_props_null <- rep(NA, B)
for(b in 1:B){
  shuffled <- sample(decisions, n)
  rand_f <- shuffled[1:n_f]
  rand_m <- shuffled[-c(1:n_f)]

  p_f_sim <-  mean(rand_f == "promote")
  p_m_sim <-mean(rand_m == "promote")

  diff_props_null[b] <- p_f_sim - p_m_sim
}
```

- Where should the null distribution be centered?

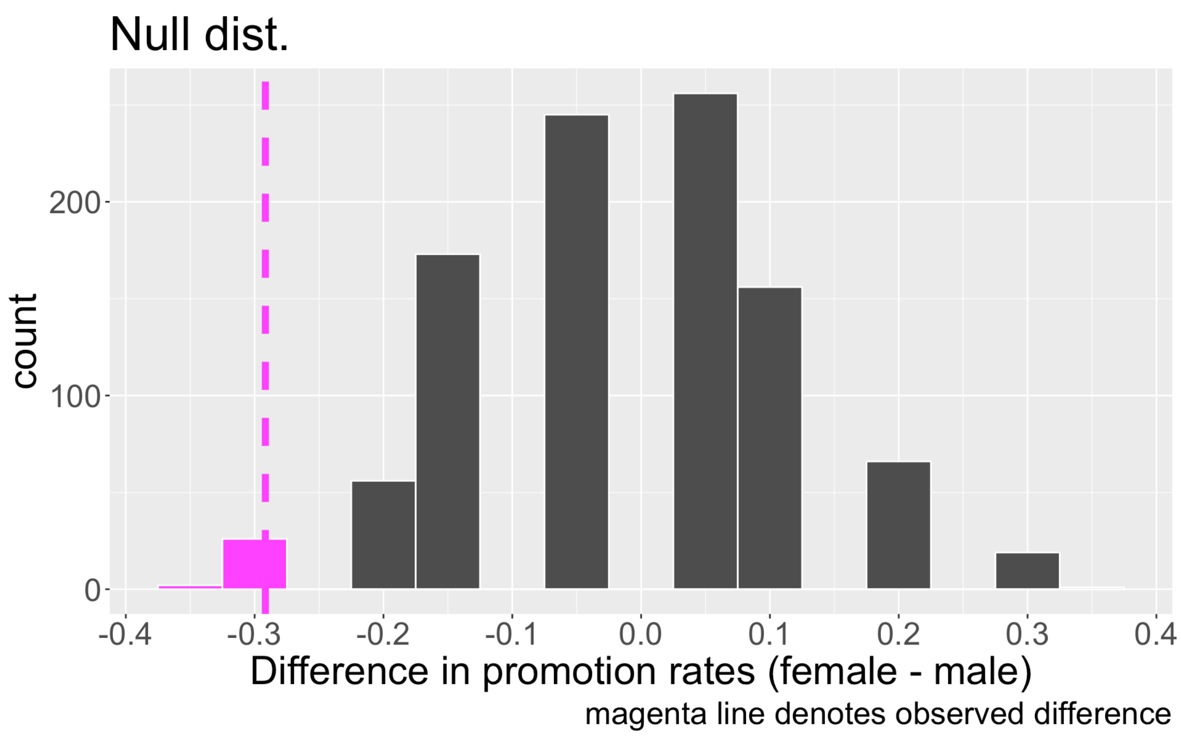
Null distribution



Obtain p-value

Recall, the observed difference in our data was $\hat{p_{f,obs}} - \hat{p_{m,obs}}$ = -0.2916667.

- p-value is probability of observing data as or more extreme than our original data, given H_0 true.
- Where does "as or more extreme" correspond to on our plot?



- Out of 1000 simulations under H_0, 28 resulted in a difference in promotion rates as or more extreme than our observed
- So the p-value is approximately 0.028

Making decision and conclusion

Our research question: Are individuals who identify their sex as female discriminated against in promotion decisions made by their managers who identify as male?

- H_0: `sex` and `decision` are independent
- H_A: `sex` and `decision` are not independent and equally-qualified female personnel are less likely to get promoted than male personnel by male supervisors
- $\alpha = 0.05$

- Interpret our p-value in context.
- Make a decision and conclusion in response to the research question.

Making decision and conclusion (answer)

- **p-value interpretation**: Assuming that `sex` and `decision` are independent, the probability of observing a difference in promotion rates as or more extreme as -0.2916667 is 0.028.
- **Decision**: Because the observed p-value of 0.028 is less than our significant level 0.05, we reject H_0.
- **Conclusion**: The data provide strong evidence of sex discrimination against female candidates by the male supervisors.
- What kind of error could we have made?

Difference in two proportions

Running example: CPR

- An experiment was conducted, consisting of two treatments on 90 patients who underwent CPR for a heart attack and subsequently went to the hospital. Each patient was randomly assigned to either:
  - treatment group: received a blood thinner
  - control group: did not receive a blood thinner
- For each patient, the outcome recorded was whether they survived for at least 24 hours.
- What is/are the variables(s) here? What types of variables are they?

Defining hypotheses

Research question: For patients who undergo CPR after a heart attack, does the blood thinner treatment have an effect on survival?

- H_0 : the blood thinner treatment has no effect on survival after heart attack
- H_A : the blood thinner treatment has an effect on survival after heart attack

Try to write down the hypotheses using statistical notation.

- Let p_T and p_C denote the proportion of patients who survive when receiving the thinner (Treatment) and when not receiving the treatment (Control), respectively

Option 1

- H_0: $p_T = p_C$
- H_A: $p_T \neq p_C$

Option 2 (preferred)

- H_0: $p_T - p_C = 0$
- H_A: $p_T - p_C \neq 0$

Collect data

Using the data, obtain the observed difference in sample proportions.

```
cpr |>
  slice(1:3)
```

```
# A tibble: 3 × 2
  group     outcome
  <fct>     <fct>
1 treatment died
2 control   died
3 control   survived
```

group	died	survived	total
control	39	11	50
treatment	26	14	40
total	65	25	90

- What evidence do we have? What summary statistic(s) would be useful for answering the research question?

Summarise data

group	died	survived	total
control	39	11	50
treatment	26	14	40
total	65	25	90

```r
# pull() takes a column from data frame and turns into vec
p_hat_c <- cpr |>
  filter(group == "control") |>
  summarise(p = mean(outcome == "survived")) |>
  pull(p)
p_hat_t <- cpr |>
  filter(group == "treatment") |>
  summarise(p = mean(outcome == "survived")) |>
  pull()
obs_diff <- p_hat_t - p_hat_c
```

- $\hat{p}_{C,obs} = \frac{11}{50} = 0.22$
- $\hat{p}_{T,obs} = \frac{14}{40} = 0.35$
- Observed difference: $\hat{p}_{T,obs} - \hat{p}_{C,obs} = 0.13$

- Is this “convincing evidence” that blood thinner usage after CPR has an effect on survival?
- Set $\alpha = 0.05$

Simulate under null

- We will once again perform a *randomization* test to try and simulate the difference in proportions under H_0
  - Under H_0, treatment group is no better than control group, so let's simulate assuming that outcome and treatment are independent

- Try filling out worksheet!

- Write down `died` on 65 cards, and `survived` on 25 cards. Then repeat several times:
  - Shuffle cards well
  - Deal out 50 to be Control group, and remaining 40 to be Treatment group
  - Calculate proportions of survival $\hat{p}_{C,sim}$ and $\hat{p}_{T,sim}$
  - Obtain and record the simulated difference $\hat{p}_{T,sim} - \hat{p}_{C,sim}$

Simulate under null (code)

Live code or look here:

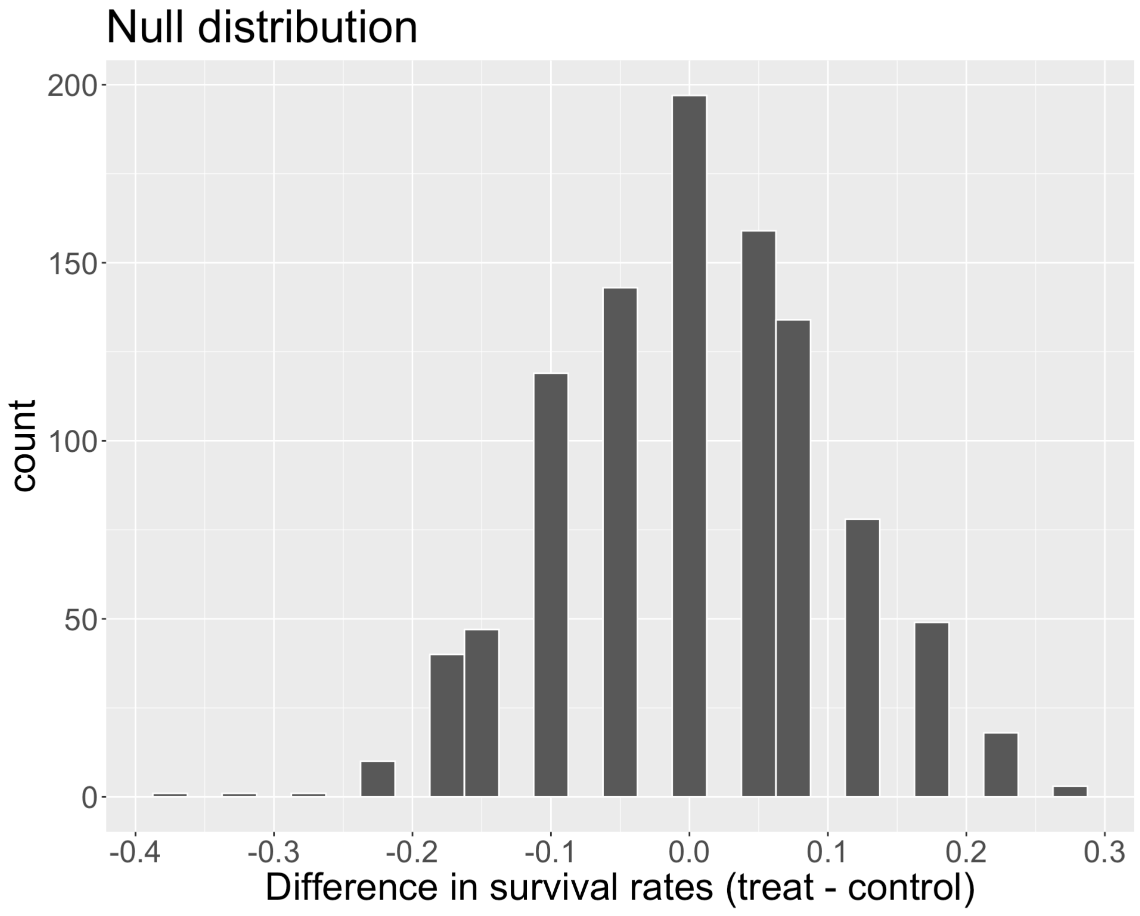
```
set.seed(310)
n_t <- sum(cpr$group == "treatment")
n_c <- sum(cpr$group == "control")
cards <- cpr$outcome
B <- 1000
diff_props_null <- rep(NA , B)
for(b in 1:B){
  shuffled <- sample(cards)
  treat_sim <- shuffled[1:n_t]
  control_sim <- shuffled[-c(1:n_t)]

  p_t_sim <- mean(treat_sim == "survived")
  p_c_sim <- mean(control_sim == "survived")

  diff_props_null[b] <- p_t_sim - p_c_sim
}
```

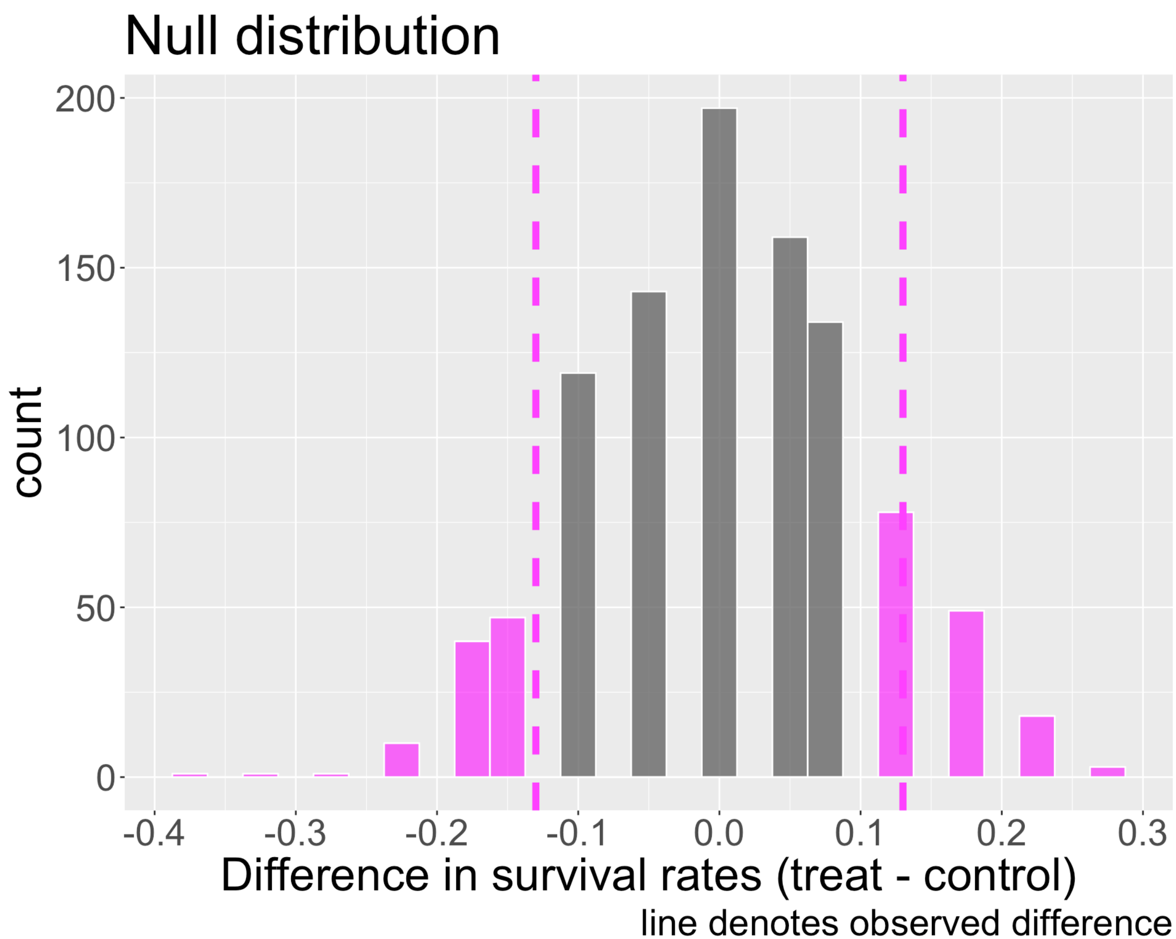
Where should our null distribution be centered at?

Visualizing null distribution



How would we obtain the p-value in this problem? What does it mean to be “as or more extreme” in the direction of H_A?

Calculate p-value



- We simulated 248 out of 1000 simulations where the difference in proportions under H_0 was as or more extreme than our observed difference of 0.13
- So p-value is approximately 0.248

Interpret and make conclusion

The researchers are interested in learning if the blood thinner treatment has an effect on survival after heart attack.

Our p-value is 0.248.

- Make a decision and conclusion about the research question in context. What type of error could we have made?

- **Decision**: because our p-value of 0.248 is greater than $\alpha = 0.05$, we fail to reject H_0
- **Conclusion**: the data do not provide convincing evidence that the blood thinner treatment affects survival rates among patients who undergo CPR.
- **Possible error**: Type 2

Comprehension questions

- What were the similarities and differences between:
  - hypothesis test for independence
  - hypothesis test for two proportions
- How do the randomization tests today differ from the test for one proportion that we learned last class?