Hypothesis testing for a mean

2025-10-23

Housekeeping

- Make sure to start working on Problem Set 6!

Recap

- We have seen how to perform hypothesis tests for questions involving the following:
  - A single proportion (Middle “berry” vs “burry”)
  - Independence of two categorical variables (banker sex discrimination)
    - Think of as one population
  - Difference in two proportions (blood thinner)
    - Think of as two populations
- We are now going to see another hypothesis test, this time for *numerical* data

Test for a single mean

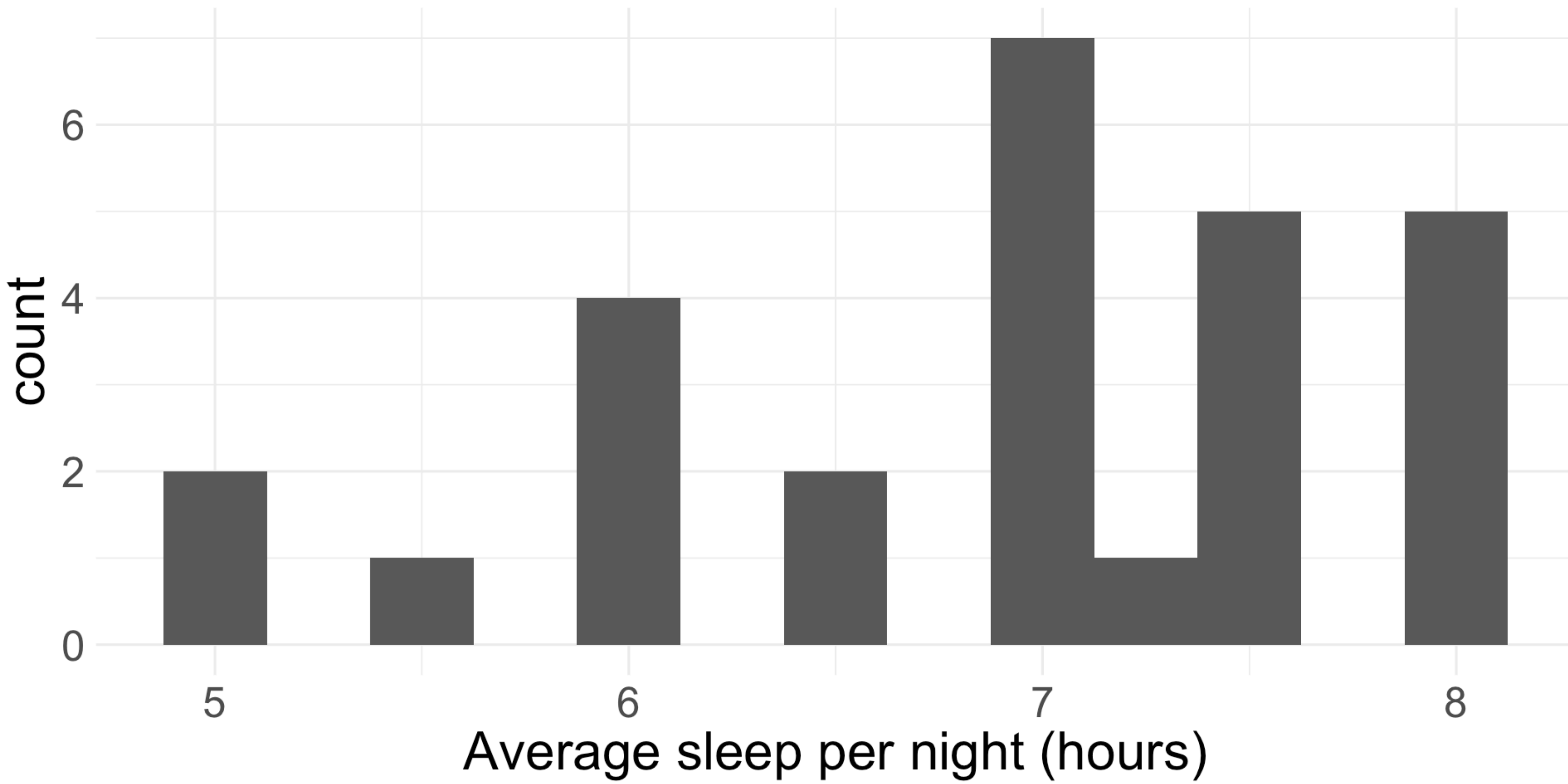
Running example + form hypotheses

We will use the the data I collected from you about average hours of sleep per night.

What type of variable(s) do we have?

- I know most college students don't receive the recommended 8 hours of sleep. But what about 7.5 hours?
- Before we look at the data, we should form our hypotheses. Suppose I am interested in learning if Middlebury students who have an 8:15am class get at most 7.5 hours of sleep (on average).
- What might our hypotheses be?
  - $H_0: \mu = 7.5$ versus $H_A: \mu < 7.5$, where μ is the true mean of the average hours of sleep Middlebury students with an 8:15 am class get.
    - Terminology: I will refer to $\mu_0 = 7.5$ as my "null hypothesized value". (i.e. the specific value of μ in H_0)

Collect and summarise data



The observed/sample mean of average hours of sleep per night is $\bar{x}_{obs} = 6.896$ from a sample of 27 students

- Now we must determine if we have "convincing evidence"! Choose $\alpha = 0.05$

Simulating null distribution

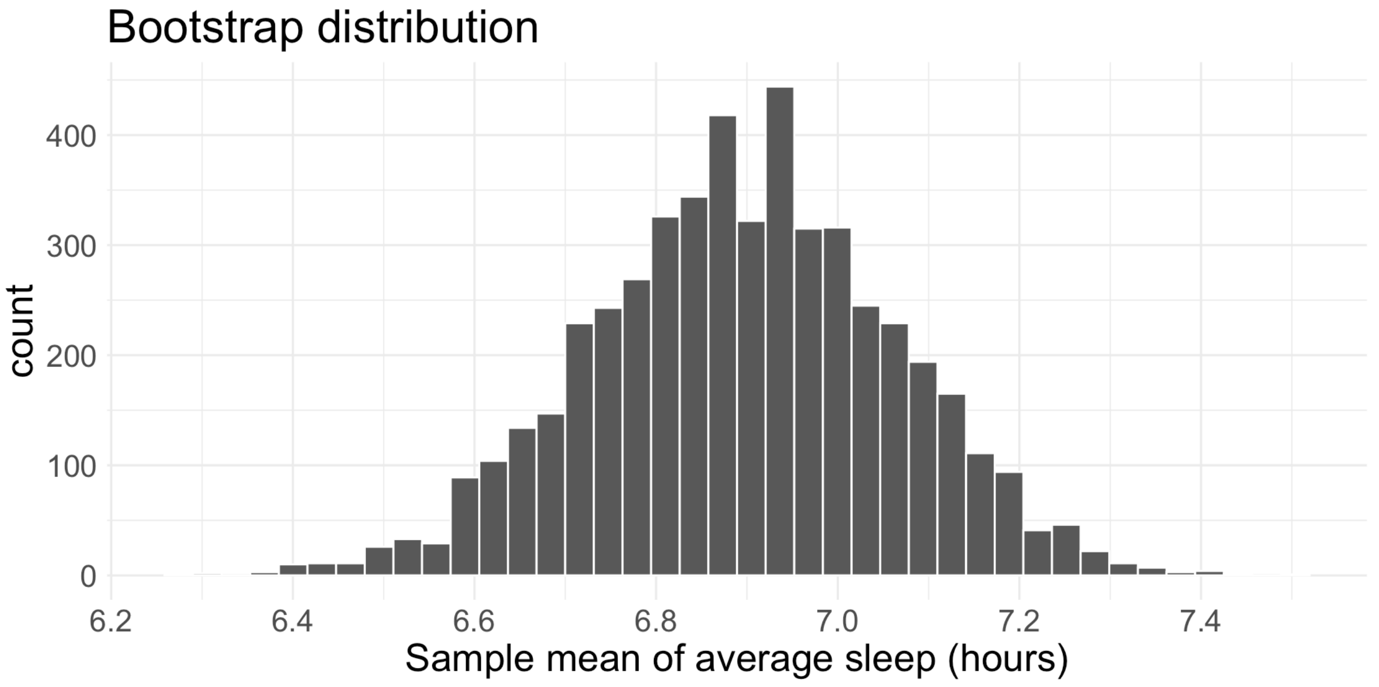
To simulate from the null distribution, we need to operate in a world where H_0 is true.

- So, I need to repeatedly simulate data sets of size 27 where true mean is $\mu_0 = 7.5$, without changing anything else
  - Specifically, my hypotheses aren’t saying anything about spread, just center!
- If I don’t want to make any assumptions about how the data behave, how might I do that?

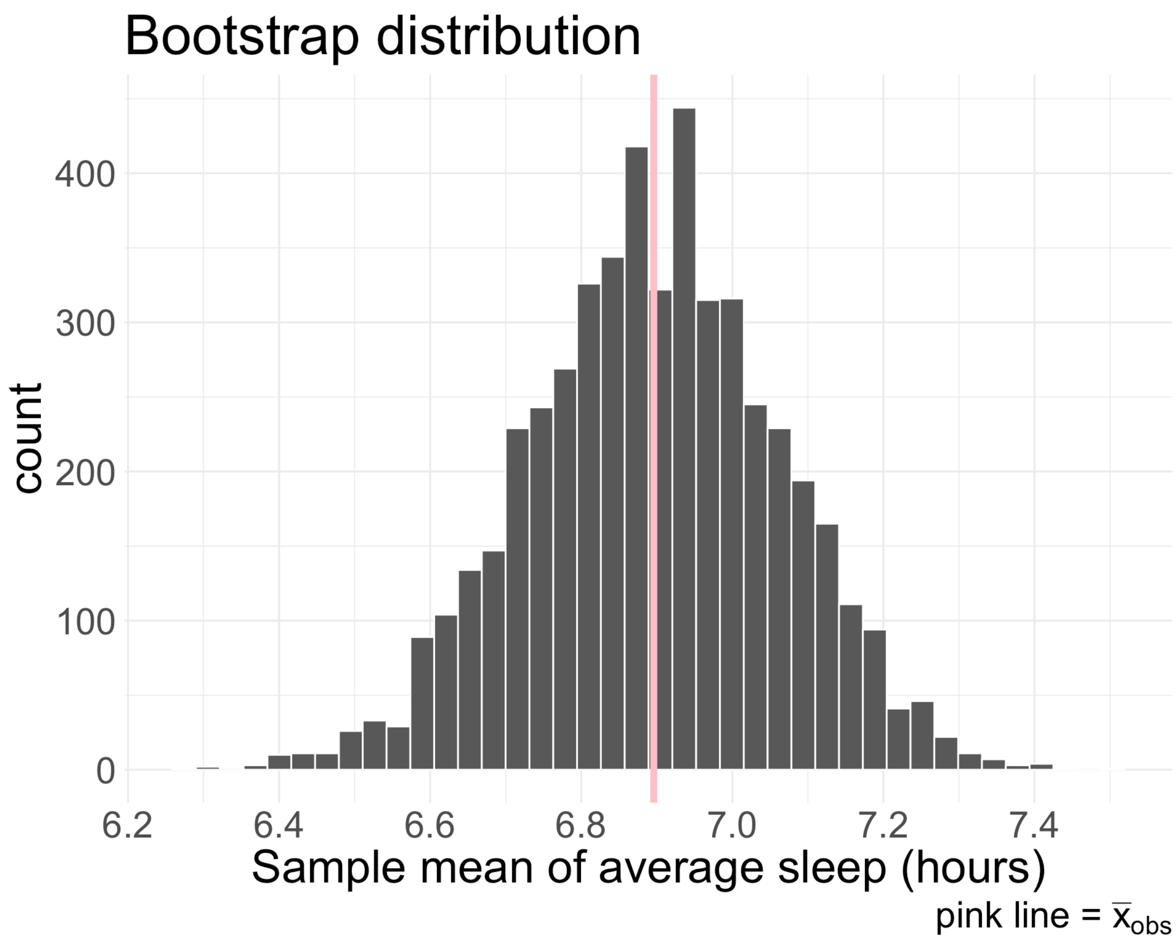
Recap bootstrap (NOT exactly the strategy)

How would I obtain a bootstrap distribution of the sample mean of mean hours of sleep?

Remind ourselves: Where should the bootstrap distribution be centered?



Bootstrap to null distribution



- This is **not** the null distribution! The null distribution should be centered at $\mu_0 = 7.5$
- However, the null distribution *should* have the same variability in $\bar{x}$ as the bootstrap distribution.

- So to get the null distribution, why not just **shift** the bootstrap distribution to be centered where we want it to be?

Shifting the bootstrap distribution

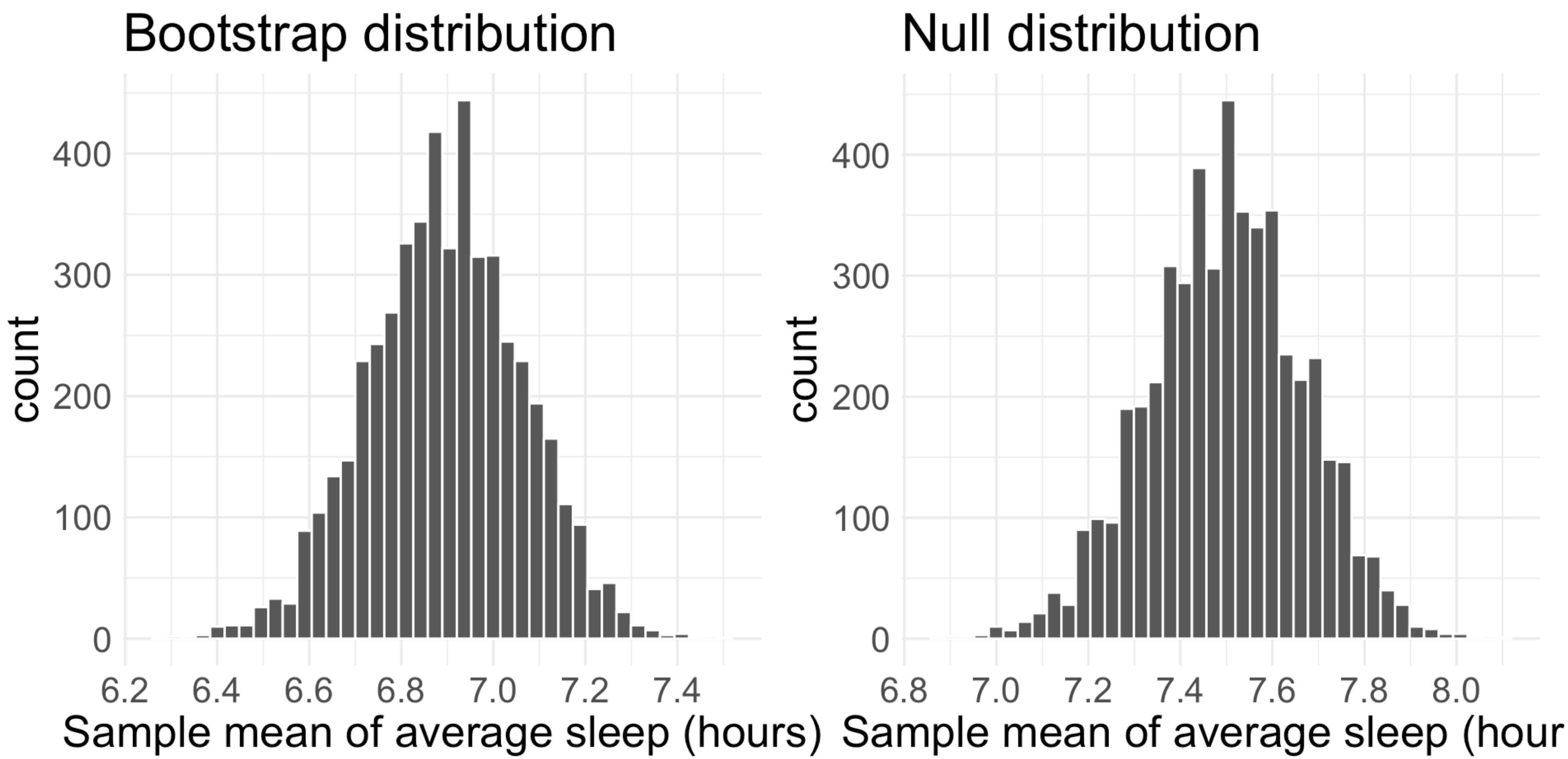
- In this example, bootstrap distribution is centered at $\bar{x}_{obs} = 6.896$
- In order to center this distribution at $\mu_0 = 7.5$, just subtract $6.896 - 7.5 = -0.604$ from every single bootstrapped mean
  - This will give us a simulated distribution for $\bar{x}$ centered at $\mu_0 = 7.5$, which is exactly the null distribution!
  - We call this "shifting the bootstrap distribution", because we simply shift where the bootstrap distribution is centered

```
mu0 <- 7.5

# xbar holds observed sample mean
shift <- xbar - mu0

# boot_means is a vector holding B bootstrapped sample means
null_dist <- boot_means - shift
```

Null distribution

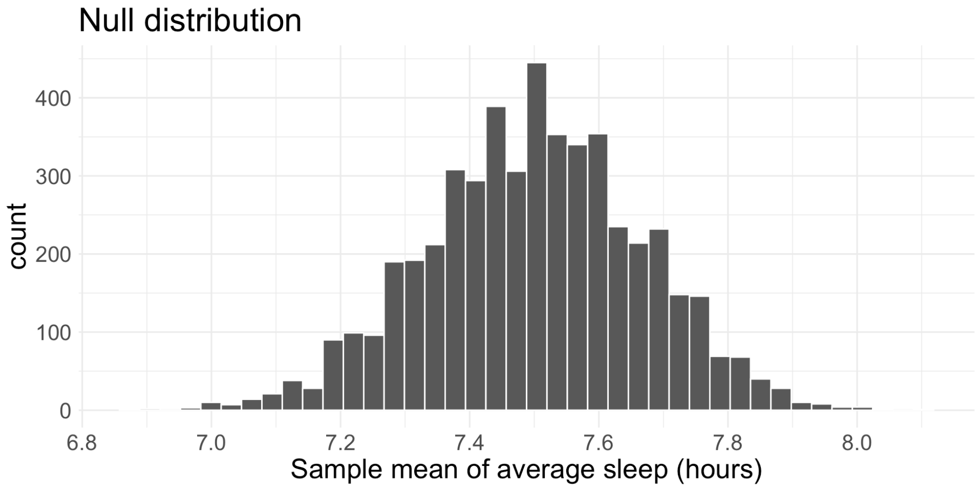


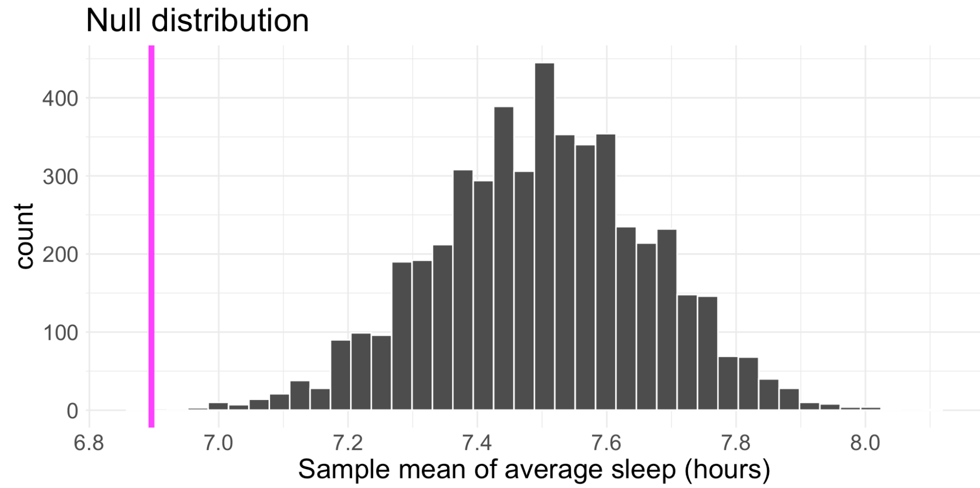
- Notice where the distributions are centered. Also note: graphs aren't exactly identical due to binning of histogram.

Obtain the p-value

$H_0: \mu = 7.5$ versus $H_A: \mu < 7.5$

Our observed sample mean is $\bar{x}_{obs} = 6.896$.





How do we find our p-value?

- In 5000 iterations, simulated 1 “as more extreme” than our data, so p-value is approximately 0.0002.

Make decision and conclusion

Make a decision and conclusion in the context of the research question.

- Since our p-value of 0.0002 is less than the significance level of 0.05, we reject H_0. We have convincing evidence to suggest that the true mean of the average hours of sleep per night that a Middlebury student with an 8:15am course receives per night is less than 7.5 hours.

Two-sided alternative hypothesis

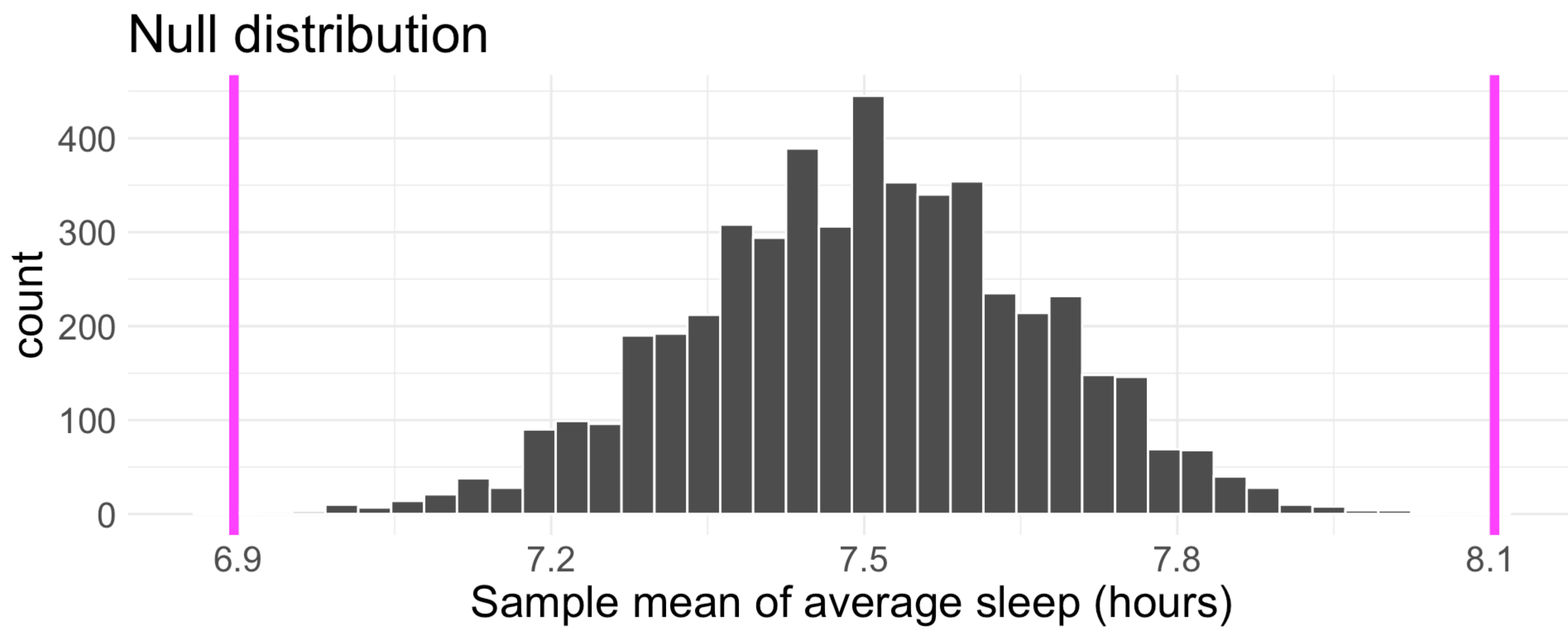
What if instead, I am interested in learning if Middlebury students who have an 8:15am class get 7.5 hours of sleep or not (on average). Then our hypotheses are:

$$H_0 : \mu = 7.5 \qquad \text{vs.} \qquad H_A : \mu \neq 7.5$$

- What does it mean to be "as or more extreme" now? Remember, $\bar{x}_{obs} = 6.896$

Two-sided alternative hypothesis

We can be extreme in **both** the positive and negative direction of μ_0!



- Note that there are now 2 simulations that are "as or more extreme", so p-value is approximately 0.0004.

Obtain the p-value (cont.)

Let $shift$ represent the amount we shifted the bootstrap distribution by:

$$shift = 6.896 - 7.5 = -0.604$$

- Simulated sample means as or more extreme as the following will contribute:
  - $\mu_0 + shift = 7.5 + -0.604 = 6.896$, or
  - $\mu_0 - shift = 7.5 - -0.604 = 8.104$
- **Note that how you code will depend on if shift is positive or negative. That's why it's important to summarise your data in step 2!**

```
sum( (null_dist <= mu0 + shift) | (null_dist >= mu0 - shift))/B
```

```
[1] 0.0004
```

Make decision and conclusion

Make a decision and conclusion in the context of the research question.

- Since our p-value of 0.0004 is less than the significance level of 0.05, we reject H_0. We have convincing evidence to suggest that the true mean of the average hours of sleep per night that a Middlebury student with an 8:15am course receives per night is **different from** 7.5 hours.

Comprehension questions

- Why did we shift the bootstrap distribution?
- Why can't we simulate null world like we did in the case of proportions?
- How does the p-value from a two-sided H_A compare to that of a one-sided H_A?
- How is this different from the homework problem where you're comparing two means?